ÉRETTSÉGI VIZSGA • 2018. május 8.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2018. május 8. 8:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. Shown below is a frequency distribution table of the body mass (rounded to the nearest integer kilogram) of 40 male university students.

mass (kg)	**53-56**	**57-60**	**61-64**	**65-68**	**69-72**	**73-76**	**77-80**
frequency	2	3	4	11	9	6	5

a) Use the midpoint of each class above to calculate both the mean and the standard deviation of the body mass of the 40 students. (The midpoint of a class is the arithmetic mean of the lower and upper limits of that particular class.)

Three "lightweight" and two "heavyweight" young men are needed to assist in a commercial. "Lightweights" may weigh no more than 64 kg, while "heavyweights" must weigh at least 77 kg.

b) How many different possible ways are there to select the five assistants, assuming all five of them are selected from among the above 40 university students?

Péter, one of the above students, earned 5 grades at his statistics course in the last semester. The median of his grades is 3, the mode is 2, the mean is 3.2. (Each grade is one of the numbers 1, 2, 3, 4, 5.)

c) Calculate the average absolute deviation of Péter's five grades from the mean of these grades.

a)	5 points	
b)	3 points	
c)	5 points	
T.:	13 points	

2. **a)** The angles of a planar quadrilateral (in degrees) form consecutive terms of a geometric progression. The common ratio of the progression is 3. Give the angles of this quadrilateral.

b) The angles of a convex polygon (in degrees) form consecutive terms of an arithmetic progression. The first term of the progression is 143, the common difference is 2. Give the number of sides of this polygon.

a)	4 points	
b)	8 points	
T.:	12 points	

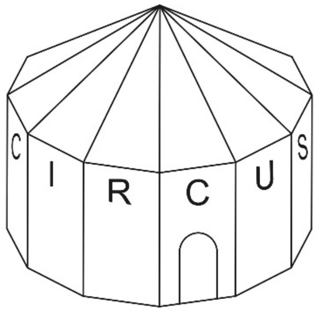
3. Solve the following inequalities in the set of real numbers.

a) $x^2 - 5x < 50$

b) $\log_3(x^2) - \log_9(81x) \leq 1$

a)	4 points	
b)	9 points	
T.:	13 points	

4. The bottom part of a circus tent is a prism with a regular 12-gon as a base. The top part is a pyramid, also with a regular 12-gon base, that fits onto the top of the prism.
The length of the base edges is 5 metres, the height of the bottom prism is 8 metres, the height of the top pyramid is 3 metres.
During winter, the tent is heated with a number of (identical) heaters, each of which is rated to heat 200 m^3.



a) What is the minimum number of such heaters required?

Titi and Jeromos are two jugglers, working for the circus. At one point in their show they are tossing clubs to each other. Both jugglers are very skilful, they only miss 3 clubs out of 1000 on average (this could also be interpreted as a probability: the probability of missing a club is 0.003). In their new show, the two jugglers plan to catch clubs a total 72 times.

b) What is the probability that they will miss no more than one club during their show? Round your answer to two decimal places.

a)	8 points	
b)	5 points	
T.:	13 points	

II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

5. **a)** Find all integer solutions of the inequality $\cos x < \frac{1}{2}$ in the interval [0; 2π].

b) How many integer solutions does the inequality $|2x-20|+|x-15|<2015$ have?

c) The function $f(x)=\left(\frac{1}{2}\right)^{x-4}-1$ is defined over the set of real numbers. How many gridpoints (points with integer coordinates) are there in the section of the first quadrant of the coordinate system bounded by the graph of function *f* and the two axes? (The borderlines also belong to this section.)

a)	3 points	
b)	8 points	
c)	5 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

6. **a)** The following are known about the non-empty sets *A*, *B*, and *C*: every element of *A* is also element of *B*, there exists at least one element of *C* that is also element of *A*. Decide whether each of the five statements below are true or false. (Proof is not required.)

(1) Set *A* has element(s) that is (are) also element(s) of set *C*.
(2) None of the elements of *C* is also an element of *B*.
(3) If something is an element of *B*, then it is also an element of *A*.
(4) If something is not an element of *B*, then it is an element of *C*.
(5) If something is not an element of *B*, then it is not an element of *A* either.

The mathematics teacher of a class of 34 students gives a short quiz at the beginning of the lesson. The quiz contains five statements and the students have to determine the truth value (true or false) of each statement. Each question is increasingly more difficult than the previous one and, consequently, they are worth more points, too. The correct answer for question n ($n \in \{1; 2; 3; 4; 5\}$) is worth n points, but if the answer is wrong, these n points are deducted. Every student, all 34 of them, answered every question.

b) Prove that there are two students whose answers are exactly the same for each question.

c) Show that the total score on this quiz must necessarily be an odd integer.

Adél, Béla, and Csilla, three students of the class, scored a total 39 points altogether on this quiz.

d) In how many different ways can 39 be written as the sum of three odd integers, neither of which is greater than 15, if the order of the terms is considered important?

a)	3 points	
b)	4 points	
c)	4 points	
d)	5 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

7. **a)** Consider the function $f(x) = ax^2 + bx + c$ ($x \in \mathbf{R}$, $a, b, c \in \mathbf{R}$ és $a \neq 0$).

Calculate the value of *a*, *b*, and *c*, if $f'(2) = 6$, $f'(6) = 2$, and $\int_0^2 f(x)dx = \frac{50}{3}$.

b) Give the equation of the line that is passing through the point *P*(0; 35) and is tangent to the parabola $y = -\frac{1}{2}x^2 + 8x + 3$.

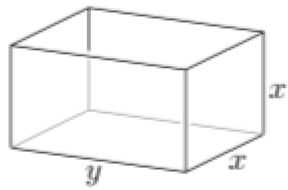
a)	7 points	
b)	9 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

8. A square cuboid (square-based straight prism) has exactly four edges that are 10 cm long. The solid diagonal of the cuboid is 12.5 cm long.

a) Calculate the surface area of the cuboid.

We have bought a fish tank that has the shape of a square cuboid. The tank is open on the top, the square faces are vertical (see diagram), and it holds exactly 288 litres of water. We would like to know whether this tank was the right choice considering unwanted algae growth on the inside of the glass walls.

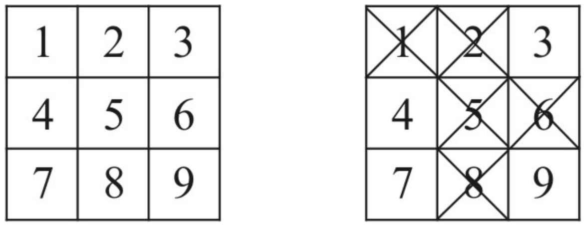


b) Calculate the length, in decimetres, of each edge (on the inside) of the tank that, while meeting all of the above conditions, provide the smallest possible interior surface area.

a)	6 points	
b)	10 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

9. Ottó is arranging a class lottery game. Five numbers will be selected out of 1, 2, 3, 4, 5, 6, 7, 8, 9 and, accordingly, five numbers should be marked on each lottery ticket, too. (Shown below is a blank ticket and also one with a possible way of filling it.)



a) András would like to get at least three of the winning numbers right with as few tickets as possible. What is the minimum number of tickets he should play with, in order to get at least three of the winning numbers right on at least one of his ticket for certain?

b) Dóra and Zoli both fill one (valid) ticket each, randomly. What is the probability that exactly four numbers on their tickets will be the same?

c) How many different ways are there to fill this class lottery ticket, so that the product of the five numbers marked is divisible by 3780?

a)	4 points	
b)	5 points	
c)	7 points	
T.:	16 points	

	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	13		51	
	2.	12			
	3.	13			
	4.	13			
Part II		16		64	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

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	pontszáma **egész számra** kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum

dátum

javító tanár

jegyző